**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

### Ans – B. 0.2676

### Since work beings 10 mins after the car is dropped, the time left to complete work is 45 mins.

### Probability that service Manager cnnot meet his commitment = P(X>50) = 1-Pr(x<=50)(X is

### the time taken to complete work.) Convert 50 to z-score

### Standard normal variable Z=(X-*μ* )/ *σ = (x-45)/8*

### *P*(X<=50)=P(Z<=(50-45)/8)=PR(Z<=0.625)=0.73237= 73.237% (the number in z-table)

### Probability that service manager will not meet his commitment is :100-73.237=26.763%=0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

### Ans –

### *μ* = 38 & *σ* =6

### a) Probability of employees>44= Pr(x>44)=1-Pr(x<=44)

### Z=(X-*μ* )/*σ* =(x-38)/6

### Pr(x<=44)=Pr(Z<=(44-38)/6)=Pr(Z<=1)=0.84134=84.134%

### Probability that employees will be greater than 44 = 100-84.134 =15.866

### Probability of employess between 38 & 44 = Pr(x<=44) – Pr(x>=38)

### Here, Pr(x<=44) = 0.84134

### Pr(x>=38)=Pr(z>=(38-38)/6)=Pr(z>=0)=0.5

### Therefore, Pr(x<=44) – Pr(x>=38) = 0.84134 – 0.5 = 0.34134 = 34.134%

### S0, the statement “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.

## b)Probability of employess less than 30 = Pr(X<30)

## Z=(X-µ)/ *σ = (30-38)/6*

## *Pr(x<30)=Pr(z(30-38)/6)=Pr(z<-a.3333)=0.09176=9.17%*

## *So,the probability of employees with probability 0.09176 of the being under*

## *30=400\*0.0917=36.68=36*

## *The statement of “employees under the age of 30 at the center would be expected to attract about 36 employees “is True.*

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(*σ* μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

2 X 1 and X 1 + X 2 ? Discuss both their distributions and parameters.

## Ans. iid = independent, identically distributed random variable

## X1 = N ( µ, σ 2 ) , X 2 ~ N ( μ, σ 2 )

## This means that the X1 and X2 are normally distributed with mean and variance. Mean

## and variance are the parameters of the of the normal distribution.

## Where −∞ &lt; µ &lt; ∞, σ 2 &gt; 0.

## Increasing mean shifts the density towards right and increasing variance flattens the

## density without shifting it.

## If X1 and X2 are 2 independent, identically distributed random variable then their sum

## and differences are also normally distributed with their means and variance being added. Means

## and variance are parameters of ND.

## X1 + X2 = N ( µ + µ, σ 2 + σ 2 )

## X1 - X2 = N ( µ - µ, σ 2 + σ 2 )

## If X1 and X2 are 2 independent, identically distributed random variable then 2 X1 will

## also be normally distributed with mean and variance being multiplied by 2.

## 2 X1 = N ( 2 µ, (2σ) 2 )

## = N( 2 µ, 4σ 2 )

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9 (-2.57 )

### Ans – D. 48.5,141.5

### The probability og getting value between a & b is 0.99

### So, the probability of getting value outside a & b is 1-0.99 = 0.01

### The probability towards left of a = -0.01/2= -0.05

### The probability towards right of b = 0.01/2= 0.05

### Since we have probabilities of a & b, we need calculated the probability of X – the random variable at a & b which has these probabilities

### By finding Standard Normal Variable (Z),need to calculated X:

### Z=(X-Mue)/Sigma

### For a probability of 0.005, z values is -2.57

### Z\**σ* + μ=x

### -(-2.57)\*20+100=151.4

### )\*20+100=48.6

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

